Last time: Stellar Motions and vital statistics
• Stars move among one another - in ways we can measure
• Motions provide more clues to stellar distance
• With distance we can determine luminosity & do a census of the stars
• Luminosity and temperature correlate - the H-R diagram

Today: Stellar Families, Masses and Luminosities
• H-RD reveals distinct groups - dominated by the Main Sequence
• Masses of stars can be found using binary star systems
• The Main Sequence is a sequence of Mass
• Mass and Luminosity correlate - the M-L relation as a consequence of fundamental physics

Features on the H-R Diagram
• The Main Sequence
  • diagonal band
  • 90% of all stars are Main Sequence stars
• The Giants
  • upper right
  • high L, low T -> huge size; 100 $R_{\text{sun}}$ and more!
• White Dwarfs
  • lower left
  • low L, ~high T -> tiny size; 0.01 $R_{\text{sun}}$ and less

Features on the H-R Diagram
An H-R diagram for all stars with Hipparcos (space) parallaxes (distance limited)
An H-R diagram for the brightest stars in the sky (brightness limited)
Main Sequence stars are the most numerous

BUT

The most prominent stars in our sky are the rare but luminous blue main sequence, giants and supergiants

• Why such variety?
• What makes stars so different from one another?
• What are we missing? MASS!

Measuring Stellar Masses: Binary Stars

• Kepler’s Third Law - for binary stars

\[
d^3 = M_1 + M_2
\]

orbital period [yrs]

• The See Saw Law

\[
\frac{M_1}{M_2} = \frac{d_2}{d_1}
\]

center-of-mass

• sum and ratio of masses allows determination of the individual masses of each star

Types of binary stars

• Visual
  • widely separated (10-100 a.u. and more)
  • know \(d_1+d_2\), \(d_2/d_1\), \(P\) (sometimes)

• Spectroscopic
  • spectral lines show periodic Doppler shifts
  • too close to see individual stars
  • know \(d_2/d_1\) (from velocities), \(P\)

• Eclipsing
  • brightness variations as stars eclipse one another
  • know \(P\), shapes of stars, light distribution

• Eclipsing spectroscopic - rare
  • provide \(d_1+d_2\), \(d_2/d_1\), \(P\) and so masses
  • radii from eclipses and orbital velocities

• Astrometric
  • stars that “wiggle”
  • bright star orbiting an unseen companion
  • provides \(d_2\), \(P\)
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- **more than 50% of stars are in binary or multiple systems**

- **BUT only a few dozen** can be used to measure accurate stellar masses

- **Key Observation:**
  Stars with the same mass have the same spectral type... on the Main Sequence

Properties of Main Sequence Stars

<table>
<thead>
<tr>
<th># in Galaxy for each O star</th>
<th>$L/L_{\odot}$</th>
<th>$M/M_{\odot}$</th>
<th>$R/R_{\odot}$</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>260,000</td>
<td>20</td>
<td>10</td>
<td>Rigel</td>
</tr>
<tr>
<td>100,000</td>
<td>60</td>
<td>3</td>
<td>2.5</td>
<td>Vega</td>
</tr>
<tr>
<td>1,000,000</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>Sun, Capella</td>
</tr>
<tr>
<td>5,000,000</td>
<td>0.06</td>
<td>0.4</td>
<td>0.6</td>
<td>Barnard’s Star</td>
</tr>
</tbody>
</table>

- **Lower mass limit** of Main Sequence: **0.08 $M_{\odot}$**
  - stars less massive don’t get hot enough to burn hydrogen

- **Upper mass limit: ~ 200 $M_{\odot}$**
  - if $M > 100 M_{\odot}$, violently unstable
**Main Sequence Extremes**

**High Mass:**
R136a1 at ~300 M_{sun}

**Low Mass:**
an ‘L’ Dwarf at 0.077 M_{sun}

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**The Mass-Luminosity Relation**

\[ L \sim M^4 \]

- Eddington (1926):
  \[ L \propto M^4 \] for main sequence stars

- Main sequence is a sequence in MASS blue stars are more massive than red stars

- The Sun is a M.S. star
  - The Sun burns hydrogen in its core

- all M.S. stars burn hydrogen in their cores

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**The Vogt-Russell Theorem (1926):**

Properties of ordinary stars are determined uniquely by mass and composition

- Mass+Composition -> position in H-R diagram
- on M.S., star burning hydrogen
- **BUT:** star is voluntarily changing its composition!
- V-R theorem demands:
  - star must leave M.S. when hydrogen is exhausted
  - so stars must move in the H-R diagram as they age
- “Stellar Evolution”
Life Expectancies for Main Sequence Stars

- available fuel supply $\propto$ mass
- rate of fuel consumption $\propto$ luminosity
- rate of consumption x lifetime = total fuel consumed
  - so... luminosity x lifetime $\propto$ mass
  - or lifetime $\propto$ mass / luminosity
- combine with luminosity $\propto$ mass$^4$ to give
  \[ t_{\text{ms}} \propto 1/M^3 \]
  \[ t_{\text{ms}} = 10^{10} \text{ yr} \times (M/M_{\text{sun}})^{-3} \]
- Massive Stars burn out faster