Newton & the "Last Word" on Orbits

Kepler's Empirical Laws
- Based solely on a simple description
- Predictions enabled, - Law -
- Not an explanation (what? not why?)

Newton
- Peculiar historical background
- Pre-calculousness
- Galileo influence, etc.

Newton's Laws (Physics 221) address motion:

Full generality - motions of point masses
- Subject to external forces
  - Separability of orthogonal directions
  1. Motion continues at constant velocity (speed + direction)
  2. Change in velocity is proportional to force,
     in same direction as force, inversely with mass

3. Action = - Reaction

Universal Gravitation
- Gravity is a universal, attractive force between bodies
- Depends solely on mass and distance:
  \[ F = \frac{G m_1 m_2}{r^2} \]

Note: \( F = ma \), so \( \ddot{a} = -\frac{F}{m} \) is grav. accel.

Auguste Auguste measured value on Earth: 9.8 m/s^2
Hold this thought:

Newton on Orbital Motion

Calculus = instantaneous change

- \( v_r \) is motion radially, towards or away from \( \Theta \)
- \( v_\theta \) = \( \Omega \), so no force effects from gravity

Ordinary body (i.e. Moon) is in perpetual free-fall

Simplest case: circular motion: no net radial acceleration
  

\[ \text{Simplest case: circular motion: no net radial acceleration} \]
Force balance = no net force = constant energy = potential energy of gravity.

\[ m \frac{GM}{r^2} = m \frac{v^2}{r} \]

\[ v^2 = \frac{GM}{r}, \quad v = \sqrt{\frac{GM}{r}} \] is circular velocity.

Orbital period \( T = \frac{2\pi r}{v} \). Only works for moon, acceleration drops with \( r^2 \).

Kepler's laws:

1. \( \frac{r^3}{T^2} = \text{constant} \)
2. \( \frac{v}{r} = \text{constant} \)
3. \( \frac{L}{r^2} = \text{constant} \)

Kepler's 3rd law: (planet to sun)

\( r \propto m^{1/3} \)

Escape velocity: \( E = 0 \)

\[ \frac{1}{2} m v^2 = \frac{GMm}{r} \]

\[ v = \sqrt{\frac{2GM}{r}} \]

Kepler 1, cont'd

Conic sections:
- ellipse: bound orbit
- parabola: \( v = v_{esc} \), total energy 0
- hyperbola: \( v > v_{esc} \), unbound

Kepler 2

Angular momentum conserved

\[ L = r \times mv \]

\( \frac{dL}{dt} = 0 \) - momenta = const.

\[ \frac{1}{2} m v^2 = \frac{L^2}{2mr^2} + \frac{GMm}{r} \]

Center of mass if one focus.
Tidal Forces

- Important for studying details of planetary/satellite interiors
- Orbital evolution via transfer of orbital energy and deformations, rotational energy, and vice-versa

Differential Gravitational Force

\[ \mathbf{F} = \frac{G M m}{R^2} \]

Force difference between A & C

\[ F_A - F_C = \frac{G M m}{R^2} \left( \frac{1}{(R-r)^2} - \frac{1}{R^2} \right) \]

\[ = \frac{G M m}{R^2} \left( \frac{R^2 - (R-r)^2}{(R-r)^2} \right) \]

\[ = \frac{G M m}{R^2} \left( \frac{2 R r - 2 R^2 + r^2}{R^2 - 2 R r + r^2} \right) \]

\[ = \frac{G M m}{R^2} \left( \frac{2 R r}{R^2 - 2 R r + r^2} \right) = \frac{G M m}{R^2} \left( 2 R \frac{r}{R^2 - 2 R r + r^2} \right) \]

Assume \( \frac{r}{R} \ll 1 \)

\[ \mathbf{a}_{\text{surf}} - \mathbf{a}_{\text{core}} = -\frac{G M}{d^2} \left[ \frac{2}{R} \cos \phi \right] + \frac{2GM}{d^2} R \cos \phi \]

\[ = -\frac{G M}{d^2} \left[ 2 \frac{R \cos \phi}{R^2} \right] + \frac{2GM}{d^2} R \cos \phi \]
Tides

\[ \Delta a = \frac{2GM_R}{d^3} \cos \phi \]

Moon tide \[ \approx 2.8 \times 10^{-8} \]

Very small!

\[ d = \frac{GM_R^3}{M} \]

But \( M = \frac{4}{3} \pi R^3 \rho \)

\[ d = \left[ \frac{2M_\odot}{4\pi \rho} \right]^{\frac{1}{3}} = \left[ \frac{2\rho \odot}{2R_\odot} \right]^{\frac{1}{3}} \]

\( d = 1.26 R \) \( (\rho_\odot/R_\odot) \)

\( \text{Wu} - \text{Roche Distance} \)

**Tides → Deformation**

(near-central effective potential)

**Tides**

- Stretching out to a football shape, pointing at companion
- If rotating non-synchronously, the bulge pushes "ahead" of direct line of centers
- Net torque pulls companion along
- Add orbital energy at expense of rotational energy
- Orbit of (moon) moves out (longer period via Kepler's 3)
- Rotation of Earth slows
- Currently, month lengthening \( \approx 0.04/\text{century} \)
- Rotation of planet slows
  - (moon already synchronous) \( \Theta \) day lengthening by \( 0.005/\text{century} \)

**Tidal bulge**

\[ \text{net effect of tidal force/accel} \]

\[ \cos \phi > 0 \]

\[ \cos \phi < 0 \]
Another effect: - @ rotation filled W.R.T. Moon orbit

- rotational flattening @ poles
- moon exerts a torque on equatorial bulge
- precession (like a gyroscope) of pole of @ rotation
- wobble period ~ 26,000 years

Tidal Heating
- small in @-moon
- much bigger in resonant systems
  (i.e. Jupiter's moons). More later