

ASTRO 342 – FALL 2006

Problem Set #1 - DUE Mon., Sept. 11

1. **Celestial Equatorial Coords.** Although we only covered these very briefly in class, these problems can be worked out with just the definitions, an understanding of the path of the Sun and offsets from the Sun. (You can check your answers on the Voyager program on the Mac Lab computers, or other planetarium programs.)

a) Estimate the Sun's right ascension (R.A.) to within half an hour on Sept. 10 by extrapolating from the autumnal equinox. Is the Sun above or below the celestial equator (i.e., at positive or negative declination)?

b) What is the approximate R.A. of a star on the meridian at sunset on the same day (again to within half an hour or so)?

c) The R.A. of Venus on Sept. 10, 2006 is about 10h 31m. Assume that Venus is on the ecliptic. About what time does it cross the meridian? Is it above or below the celestial equator?

2. **Astronomical Geometry**

a) Mercury's orbital radius is about 0.387 astronomical units. What is the angle on the sky between the Sun and Mercury at greatest elongation?

b) Is it possible to see an inferior planet close to the meridian at midnight? Why or why not?

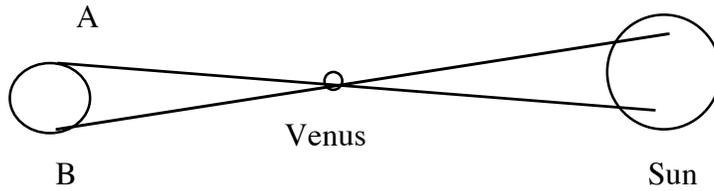
c) What is the phase (as in phases of the moon) of an inferior planet at greatest elongation? Make a sketch to illustrate your answer.

3. **Synodic and sidereal periods** - The sidereal period of a planet, P , is the time required for it to complete one trip around its orbit relative to the stars. Thus, it is the true physical period as seen from an external reference frame. The synodic period S is the time required for the planet to return to some particular alignment as seen from Earth. For example, for a superior planet the synodic period might be measured as the time between two successive oppositions. Sidereal period is used in calculations of planetary orbits, but synodic periods are historically the observed quantity. These two periods of a planet are related by the following equation,

$$\frac{1}{P} = \frac{1}{P_{Earth}} \pm \frac{1}{S_{syn}}$$

where $P_{\text{Earth}} = 1.0$ yr., and the plus + sign is used for inferior planets, while the – sign is for superior planets. (Physically, this equation says that the synodic *frequency* of a planet equals the sum or difference of its sidereal frequency and the Earth’s sidereal frequency.)

- a) The orbital periods of Jupiter and Mars are 11.86 yrs. and 1.88 yrs., respectively. Suppose that at a certain time the two planets are both at superior conjunction. What is the time interval ($\Delta t = t_2 - t_1$) for the next superior conjunction for each? (Assuming that they hit conjunction at the same time to within 0.01 yr.)
- b) How long until they are simultaneously at superior conjunction again?
- c) The synodic period of the Moon is 29.53 days. How would you observe this? What is its sidereal period?
4. **Finding the A.U.** (and the scale of the solar system). The classical greeks estimated the relative distances of the Moon and the Sun, but not absolute values. Although we now know the A.U. (the mean radius of the Earth’s orbit) to within about a meter, historically it was very hard to determine. You might think we could just use Newton’s general form of Kepler’s third law, but we don’t know the mass of the Sun a priori. In fact, it is better to turn this around by determining the A.U. independently, and using the general Kepler third law to calculate the mass of the Sun.
- a) The greatest elongation of Venus is observed to be about 47 degrees. Assume Venus’ orbit is circular. How much closer to the Sun is Venus than Earth? That is, what is the ratio of their orbital radii?
- b) The period of Venus can be determined by observing the time between successive inferior conjunctions. This time interval is found to be 1.60 yrs. What is the orbital period of Venus? (Hint: remember the previous problem.)
- c) The key step is to measure the so-called transit of Venus, i.e., observe Venus at an inferior conjunction when the alignment is such that Venus actually passes in front of the disk of the Sun. This doesn’t happen very often, but it was observed in the eighteen century, albeit with some loss of life in expeditions to the far corners of the world. The idea is shown by the diagram below.



This diagram shows the transit being observed from 2 points (A and B) on the Earth. The distance between the points is supposed to be known. Suppose it is about 5000 km. The angular position of the transit is accurately measured at each site, so the small angle A-Venus-B can be determined. Then the distance from Earth to Venus can be determined from trigonometry. Supposing that this distance is 0.277 a.u., how accurate would the angular observations have to be to derive this distance? **(Extra credit)** Do you think the distance could have been determined to 2 or 3 significant digits in the 1760s? What would the difficulty be?

d) The modern method of determining the distance to Venus would be to bounce a powerful radar pulse off Venus, record the travel time of the pulse, and multiply that time by the speed of light to get the distance. What is that travel time when Venus is at inferior conjunction?

Footnote: Venus transited in June, 2004, for the first time in more than 100 yrs.. (For further info. see the 5 articles of D.J. Fernie in American Scientist magazine in the late 1990s.)