Astro 342 Fall 2006 Homework #3

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(1.) Pressure scale heights for
- Earth, Venus, Mars, Pluto, Titan (surface)
- Jupiter, Neptune (1 bar)

$$\text{Pressure Scale Height} = \frac{kT(z)}{g(z)\,\mu(z)\,M_{amu}}$$

$k = 1.38\times10^{-23}$ J/K

$M_{amu} = 1.66\times10^{-27}$ kg

$g = \frac{GM}{r^2}$; $G = 6.67\times10^{-11}$ N m^2 kg^{-1}

$\mu_{H_2} = 2$ $\mu_{CO_2} = 44$

$\mu_{N_2} = 28$ $\mu_{O_2} = 32$

$\mu_{He} = 4$ $\mu_{CH_4} = 16$

$$\boxed{\lambda_p = 7.69\,\text{km}\left(\frac{T}{273\,K}\right)\left(\frac{M}{M_\oplus}\right)^{-1}\left(\frac{R}{R_\oplus}\right)^{2}\left(\frac{\mu}{30}\right)^{-1}}$$

$T \rightarrow$ p. 295 $M, R \rightarrow$ p. 392-395

Earth $T = 281$ K $\mu \sim 30$ $\rightarrow \lambda_p = 7.91$ km

Venus $T = 730$ K $\mu = 44$ $M = 0.81$, $R = 0.95 \rightarrow \lambda_p = 15.62$ km

Mars $T = 215$ K $\mu = 44$ $M = 0.107$ $R = 0.533 \rightarrow \lambda_p = 10.96$ km

Pluto $T = 37$ K $\mu = 16$ $M = 0.0022$ $R = 0.180 \rightarrow \lambda_p = 28.77$ km

Titan $T = 95$ K $\mu = 28$ $M = 0.023$ $R = 0.404 \rightarrow \lambda_p = 20.35$ km

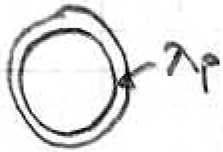
Jupiter $T = 120$ $\mu = 2.0$ $M = 317.7$ $R = 11.19 \rightarrow \lambda_p = 19.98$ km

Neptune $T = 69$ $\mu = 2.5$ $M = 17.06$ $R = 3.88 \rightarrow \lambda_p = 20.58$ km

All are strikingly similar!

2: Mass of $\oplus$ atmosphere & Venus atmosphere:

(3)



$M = 4\pi r^2 \bar{\rho}(\lambda_p \times n)$ where n is the # of pressure scale heights over which $\bar{\rho}$ is valid

$\lambda_p = 8\times10^3\ m$

$P_{surf} = 1\ bar = 10^5\ N/m^2$

$P = \rho g h\ ;\ \rho = P/gh$

$10^5 = \rho \cdot 10\ m/s^2 \cdot 8\times10^3 m \rightarrow \rho = 1.25\ kg/m^3$

Earth: $M_{atmo.} = 4\pi(6.378\times10^6)^2 \cdot 1.25 \cdot 8\times10^3\ kg$

$= \underline{5.11\times10^{18} kg} = \underline{8.5\times10^{-7} M_\oplus}$

Venus
$$\left.\begin{aligned} P &= 95\ P_\oplus \\ R &= 0.95\ R_\oplus \\ \lambda &= 1.97\ \lambda_\oplus \\ g &= 0.898\ g_\oplus \end{aligned}\right\} \quad M_{\substack{venus\\atm}} = 5.11\times10^{18} \cdot \left(\frac{R}{R_\oplus}\right)^2 \left(\frac{P}{P_\oplus}\right)\left(\frac{g}{g_\oplus}\right)^{-1} \cancel{\left(\frac{\lambda}{\lambda_\oplus}\right)^{-1}} \cancel{\left(\frac{\lambda}{\lambda_\oplus}\right)}$$

$= \underline{4.88\times10^{20} kg}$

$= \underline{1.00\times10^{-4} M_V}$

Earth + H_2O

$\rho = 1000\ kg\ m^{-3}$

$M_{H_2O} = 4\pi R_\oplus^2 \cdot 3\times10^3 km \times 1000 = \underline{1.53\times10^{21} kg}$

$= \underline{2.56\times10^{-4}\ M_\oplus}$

So Venus atmosphere mass comp. w/ H_2O on $\oplus$!

(3.) Dry air $80\% N_2,\ 20\% O_2$

(2) $\mu = 0.8 \times 28 + 0.2 \times 32 = 28.8$ amu

H_2O: $\mu = 16$ amu

$$\mu_{total} = (0.8-\epsilon) \times 28 + (0.2-\eta) \times 32 + (\epsilon+\eta) \times 16$$

$$= 28.8\text{amu} - (\epsilon+\eta) \cdot 28.8 + 16(\epsilon+\eta)$$

$$= 28.8\text{amu} - 12.8(\epsilon+\eta)$$

if $(\epsilon, \eta) > 0$ then $\mu\downarrow$

$P = \frac{\rho}{\mu} N_A k T$, so if P is constant, a decrease in μ means ρ must decrease

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(5.) (a) 50% absorption of optical light:

(3) $I(t) = I(0)\ e^{-\tau}$

$0.5 = e^{-\tau} \longrightarrow \tau = 0.693$

(b) $R_{dust} = R_{Mercury} = 0.387\text{au} = 5.81 \times 10^{10}$ m $\quad A_b = 0.5$, $\epsilon = 1$

$$\frac{L_\odot}{4\pi(5.81 \times 10^{10}\text{m})^2} \cdot \frac{1}{2} = \sigma T^4; \quad T^4 = \frac{2 \times 3.86 \times 10^{26}}{4\pi(5.81 \times 10^{10})^2\ 5.67 \times 10^{-8}}$$

$T = 532$ K

(6.) $T_e = \frac{0.29\,cm}{\lambda_{max}}$; $\lambda_{max} = \frac{0.29\,cm}{T}$

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5800 K; $\lambda_{max} = 5\times10^{-5}$ cm = 500 nm (optical)

3×10^6 K: $\lambda_{max} = 9.67\times10^{-8}$ cm = 0.97 nm (X-ray)

(7.) $I_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2}\left(e^{h\nu/kT} - 1\right)^{-1}$

2

$\frac{B_\nu(4000)}{B_\nu(5800)} = \frac{e^{h\nu/5800k} - 1}{e^{h\nu/4000k} - 1}$ $\quad \nu = \frac{c}{\lambda}$

$\frac{hc}{k} = 0.0144$

$= \frac{e^{hc/(\lambda k\, 5800)} - 1}{e^{hc/(\lambda k\, 4000)} - 1}$

$\lambda = 5.5\times10^{-7}$ m → ratio = $\frac{90.3}{695} = 0.13$

$\lambda = 10^{-6}$ m → ratio = $\frac{10.97}{35.60} = 0.31$

(8.) $L = 4\pi r^2 \sigma T^4$

photosphere: $T = 5800$K, $R = 7\times10^8$ m $\Big\}$ $L = 3.95\times10^{26}$ W ("true" value)

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corona: $T = 2\times10^6$ K, $R = 1.4\times10^9$ m $\Big\}$ $L = 2.23\times10^{37}$ W (!)

clearly, the corona is 1 - not a perfect radiator
2 - optically thin
3. not a blackbody and
the formula is not applicable

3:5

9. (a) $T_{eq} = \left(\frac{F_{sun}}{1} \frac{(1)}{4\sigma} \right)^{1/4} = \underline{278\ K}$

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(b) in this case, $\epsilon = 0.5$; $T_{eq} = \left(\frac{F_\odot}{1} \frac{1}{2\sigma} \right)^{1/4} = \underline{331\ K}$

10. 10^{25} J in 3600 s $\rightarrow 2.78 \times 10^{21}$ W

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5×10^{24} J in 1 KeV X-rays

(a) X-ray luminosity, for that hour, is 1.39×10^{21} W, 5 orders of magnitude smaller than L_{sun}

(b) $F_{astronaut} = 5 \times 10^{24}\ \text{J} / 2\pi d_n^2 \times 1\ \text{m}^2$

Earth: 35.5 J @ $d = 1.5 \times 10^{11}$ m

Mercury: 237.4 J @ $d = 5.8 \times 10^{10}$ m

(c) 1 keV $= 1.6 \times 10^{-16}$ J, so dose is

Earth $= 2.22 \times 10^{17}$ photons / m^2 = 2.22×10^{13} /cm^2 = 22 rad } ouch!

Mercury $= 1.48 \times 10^{18}$ " / m^2 = 1.48×10^{14} /cm^2 148 rad }