

ASTRO 342 – FALL 2006

Problem Set #4 - DUE Mon., Oct. 30

Do the following problems from the text –

Ch. 8: probs.: 12, 13

Ch. 9: probs.: 2, 12

1. (*This problem is a modified version of prob. 8.15 from Hartmann, Moons & Planets, 4th ed.*)

The specific heat c of basaltic rock is the amount of energy involved in a temperature change of 1K for a cubic meter of rock: it is about $c_v = 4 \times 10^6 \text{ J}/(\text{m}^3\text{K})$. (Note: the value given in Hartmann is crazy, the present value is derived from de Pater and Lissauer, pg. 248.) Suppose a magma ocean starts at time $t=0$ with a mean temperature 1800K and cools with no significant input of additional heat from below or above. The ocean is 3000 km deep and has a surface crust that radiates at a mean temperature of $T=900\text{K}$.

- a) Make an estimate of the time scale for cooling of the mean ocean temperature by 400K.
- b) Give some physical complications that require a more exact calculation in order to derive the complete cooling history from an initial molten state to today's conditions.

The following problems are from the de Pater and Lissauer text, though you don't need that text to do them.

- 5.5. If the lithospheric plates move, on average, at a speed of 6 cm yr^{-1} , what would be a typical recycling time of the terrestrial crust? (Hint: calculate the recycling time based on the motion of one plate over the Earth's surface; how would your answer change if you have several plates moving over the surface?)
- 5.8. a) Calculate the kinetic energy and pressure involved when the Earth gets hit by a stony meteoroid ($\rho = 3.4 \text{ g cm}^{-3}$) which has a diameter of 10 km, and which has a zero velocity at a very large distance from Earth.
- b) Calculate the kinetic energy involved were the same meteoroid to hit Jupiter instead of Earth, assuming the body has zero velocity at a large distance from Jupiter.

c) Calculate the kinetic energy and pressure involved when a fragment of Comet D/Shoemaker-Levy 9 ($\rho = 0.5 \text{ g cm}^{-3}$, $R = 0.5 \text{ km}$) hits Jupiter at the planet's escape velocity.

d) Express the energies from a)-c) in magnitudes on the Richter scale, and compare these with common earthquakes. Note: energy and Richter magnitude are related by the equation - $\log_{10}E = 12.24 + 1.44 M_R$.

- 6.2.** Assume that material can be compressed significantly if the pressure exceeds the material strength. If material can be compressed considerably over a large fraction of a body's radius, the body will take on a spherical shape (the lowest energy state of a nonrotating body fluid body).

Calculate the minimum radius of a rocky body to be significantly compressed at its center. Assume an uncompressed density of $\rho = 3.5 \text{ g cm}^{-3}$, and material strength $S_m = 2 \times 10^9 \text{ dyne cm}^{-2}$.

- 6.3.** Use the equation of hydrostatic equilibrium to estimate the pressure at the center of the Moon, Earth and Jupiter. Take the simplest approach, and approximate the planet to consist of one slab of material with thickness R , the planetary radius. Assume the gravity $g_p(r) = g_p(R)$, and use the mean density $\rho(r) = \rho_{\text{mean}}$.

- 6.22. (extra credit)** What is the current rate at which energy is being generated within the Earth via radioactive decay? You may assume that the Earth is made of chondritic material in which the radioactive energy generation rate is $4.0 \times 10^{-8} \text{ ergs g}^{-1} \text{ s}^{-1}$. State your answer in ergs per year. Compare this energy with the average energy released in earthquakes per year, which is $\approx 10^{28} \text{ ergs}$, and comment.

- 6.23.** Mercury's mean density is $\rho = 5.44 \text{ g cm}^{-3}$. This value is very close to the planet's uncompressed density. If Mercury consists entirely of rock ($\rho = 3.3 \text{ g cm}^{-3}$) and iron ($\rho = 7.95 \text{ g cm}^{-3}$), calculate the planet's relative abundance of iron by mass.