Distance as a pivotal quantity

Stellar Motions
- Proper motion and tangential velocity
- Radial Velocity via the Doppler Effect

Proper motion and tangential velocity

Statistical Parallax: motion of the sun for a bigger baseline

Uses of the Doppler Effect

The Hertzsprung-Russell (H-R) Diagram
- Classification tool: luminosity plotted vs. temperature
- Radius on the H-R diagram

H-R Diagram Features:
- The Main Sequence, Red Giants, and White dwarfs
- 90% of stars are on the Main Sequence

Features on the H-R Diagram

Main Sequence stars are the most numerous

BUT

The most prominent stars in our sky are the rare but luminous blue main sequence, giants and supergiants

- Why such variety?
- What makes stars so different from one another?
- What are we missing? MASS!
Measuring Stellar Masses: Binary Stars

- **Kepler’s Third Law** - for binary stars

- **The See Saw Law**

  \[
  \frac{M_1}{M_2} = \frac{d_2}{d_1}
  \]

  sum and ratio of masses allows determination of the individual masses of each star

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Types of binary stars

- **Visual**
  - widely separated (10-100 a.u. and more)
  - know \( d_1 + d_2 \), \( d_2/d_1 \), \( P \) (sometimes)

- **Spectroscopic**
  - spectral lines show periodic Doppler shifts
  - too close to see individual stars
  - know \( d_2/d_1 \) (from velocities), \( P \)

- **Eclipsing**
  - brightness variations as stars eclipse one another
  - know \( P \), shapes of stars, light distribution

- **Eclipsing spectroscopic - rare**
  - provide \( d_1 + d_2 \), \( d_2/d_1 \), \( P \) and so masses
  - radii from eclipses and orbital velocities

- **Astrometric**
  - stars that “wiggle”
  - bright star orbiting an unseen companion
  - provides \( d_2 \), \( P \)
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- more than 50% of stars are in binary or multiple systems
- BUT only a **few dozen** can be used to measure accurate stellar masses

- **Key Observation:**
  Stars with the same mass have the same spectral type... **on the Main Sequence**

### Properties of Main Sequence Stars

<table>
<thead>
<tr>
<th># in Galaxy for each O star</th>
<th>( \frac{L}{L_{\odot}} )</th>
<th>( \frac{M}{M_{\odot}} )</th>
<th>( \frac{R}{R_{\odot}} )</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>260,000</td>
<td>20</td>
<td>10</td>
<td>Rigel</td>
</tr>
<tr>
<td>100,000</td>
<td>60</td>
<td>3</td>
<td>2.5</td>
<td>Vega</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>Sun, Capella</td>
</tr>
<tr>
<td>5,000,000</td>
<td>0.06</td>
<td>0.4</td>
<td>0.6</td>
<td>Barnard’s Star</td>
</tr>
</tbody>
</table>

- **Lower mass limit** of Main Sequence: 0.08 \( M_{\odot} \)
  - stars less massive don’t get hot enough to burn hydrogen

- **Upper mass limit:** \(~ 200 M_{\odot} \)
  - if \( M > 100 M_{\odot} \), violently unstable
Main Sequence Extremes

**High Mass:**
R136a1 at \( \sim 300 M_{\text{sun}} \)

**Low Mass:**
an ‘L’ Dwarf at \( 0.077 M_{\text{sun}} \)

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The Mass-Luminosity Relation

\[ L \sim M^4 \]

- **Eddington** (1926): \( L \propto M^4 \) for main sequence stars

- Main sequence is a sequence in **MASS** blue stars are more massive than red stars

- The Sun is a M.S. star
  - The Sun burns hydrogen in its core

- all M.S. stars burn hydrogen in their cores
The Vogt-Russell Theorem (1926):
Properties of ordinary stars are determined uniquely by mass and composition

- Mass + Composition -> position in H-R diagram
- on M.S., star burning hydrogen
- **BUT**: star is voluntarily changing its composition!
- V-R theorem demands:
  - star must leave M.S. when hydrogen is exhausted
  - so stars must move in the H-R diagram as they age

- “Stellar Evolution”

Life Expectancies for Main Sequence Stars

- available fuel supply $\propto$ mass
- rate of fuel consumption $\propto$ luminosity
- rate of consumption x lifetime = total fuel consumed
  - so... luminosity x lifetime $\propto$ mass
    - or lifetime $\propto$ mass / luminosity
  - combine with luminosity $\propto$ mass\(^4\) to give
    \[
    t_{\text{ms}} \propto \frac{1}{M^3}
    \]
    \[
    t_{\text{ms}} = 10^{10} \text{ yr} \times \left(\frac{M}{M_{\text{sun}}}\right)^{-3}
    \]
- Massive Stars burn out faster