Consequences of the Equivalence Principle

\[ \gamma = \frac{1}{\sqrt{1 - \frac{r_s}{d}}} \quad r_s = \frac{2GM}{c^2} = 3 \text{ km} \left( \frac{M}{M_\odot} \right) \]

- Gravity bends light = warps space-time by factor \( \gamma \)
- time slows down where gravity is strong
  - \( t_{\text{outside}} = t \times \gamma \)
- light gets redshifted moving away from gravity source
  - \( \lambda_{\text{outside}} = \lambda \times \gamma \)
- moving masses create ripples in space-time (gravitational radiation)

Tests of GR:

- Tests of GR:
  - Bending of light by gravity

  - displaced stars near limb of Sun - the 1919 and 1923 solar eclipses
  - Gravitational Lensing

- surface of the Sun: \( d = 2.3 \times 10^5 \) \( r_s \) \( \gamma = 1 + 2.14 \times 10^{-6} \)
- “ of a white dwarf: \( d = 2.3 \times 10^3 \) \( r_s \) \( \gamma = 1 + 2.14 \times 10^{-4} \)
- “ of a neutron star: \( d = 3.3 \) \( r_s \) \( \gamma = 1 + 0.20 \)
- as \( d \to r_s \), \( \gamma \to \infty \)
  - INFINITE time dilation (time stops)
  - INFINITE redshift (wavelength increases to beyond radio)
  - INFINITE space curvature (no escape)
  - a BLACK HOLE
Shapiro Time Delay

- time for light to move along curved path is longer
  - Radar / spacecraft measurements within solar system
  - Binary pulsars

![Graph showing Shapiro time delay vs calendar date](image1)

The Binary Pulsar

Hulse / Taylor 1993 Nobel Prize

- Pulsar (clock!) in a binary
- orbit period = 7h 45m

- neutron star - relatively large γ
  - large Shapiro time delay
  - large perihelion precession rate
  - orbital decay from gravitational radiation
  - (now) 30+ years of data

![Graph showing period change from gravitational radiation](image2)

period change from gravitational radiation produces shift in periastron time

![Graph showing cumulative shift of periastron time](image3)

A binary system of compact massive objects orbiting each other produces Shapiro delay

**Consequences of the Equivalence Principle**

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**the “photon sphere”**

- **event horizon:**
  - \( v_{\text{esc}}^2 = c^2 \sim 2GM/r \)
  - escape velocity = \( c \)
- prior to (above) the event horizon:
  - \( v_{\text{orb}}^2 = c^2 \sim GM/r \)
  - orbital velocity = \( c \)
  - \( r_{\text{photon}} = 1.5 \times r_s \)
- at this photon sphere
  - tangential light “orbits”
  - light directed down: spirals into BH
  - light directed upwards: spirals away

**Approaching a black hole**

- pre-encounter (\( d > 1.5 \) \( r_s \))
  - horizontally directed light curves downwards
  - perceived horizon bends upwards
  - the “bowl effect”
  - look “down” and see a bright ring at the photon sphere (“Einstein ring”)
Approaching a black hole

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- at the photon sphere
  - bowl effect very pronounced
  - straight ahead you see the back of your head!

- continue down (between P.S. and event horizon)
  - light may escape, depending on direction
  - escape cone (above) narrows on approach to $r_s$
  - outside of escape cone, view around corners
to a neutron star

- “painted” with map of Earth
- orbit
- land
- look up
- orbit at surface

to an ultracompact star

- “painted” with map of Earth
- orbit
- to photon sphere
- look up @ p.s.
- orbit
- to surface
- survey
- orbit
to an ultracompact star

- “painted” with map of Earth
- orbit
- to photon sphere
- look up @ p.s.
- orbit
- to surface
- survey
- orbit

what lies beneath?

- non-rotating: \( d < r_s \)
  - fall @ speed of light to form a singularity
  - viewed from afar, collapse stops at event horizon

- Cosmic Censorship
  - time stops \( \rightarrow \) loses meaning \( \rightarrow \) is irrelevant!
  - cut off from the rest of the Universe (almost) forever

rotating black holes

“everything” rotates

- rotation “drags” space-time along
- effect is to pull stuff in direction of BH spin
- extended capture region beyond \( r_s \)
  - “frame dragging”

anatomy of a rotating (Kerr) black hole

- static limit
  - forced co-rotation
  - even light is forced to co-rotate w/ BH

- outer horizon
  - frame dragged faster than light
  - swap of time and space
  - singularity in time, not space
  - escape possible depending on initial entry

- inner horizon
  - return to normal time/ space
fast rotation $\Rightarrow$ ring singularity

- nested “Event Horizons” surrounding the ring
- pass through ring to encounter exo-EH
- return to 3+1D Universe with more(!) energy
  - for some trajectories, anyway

Black holes have no hair

- **Black holes have**
  - MASS (but no “surface”)
  - SPIN
  - CHARGE

- **Black holes DO NOT have**
  - antimatter / matter
  - color
  - smell
  - taste
  - texture
  - information
  - “hair”

Perils of visiting a black hole

- **Tidal forces**: differential pull of gravity
  - $F_{\text{grav}} \propto 1/d^2$ so $F_{\text{tide}} \propto 1/d^3$

- normal tides are tiny:
  - lunar tide = pillow on your head / $10^{12}$

\[
\frac{\text{stretch}}{\text{weight}} = 10^7 \times \text{height} [m] \times \left[ \frac{d}{R_{\text{Sch}}} \right]^3 \times \left[ \frac{M}{M_{\odot}} \right]^2
\]

- **Solar mass Black Hole**:
  - stretch = weight @ $d = 3650$ km ($\sim 1200 R_{\text{sch}}$)
  - “$\sim 10 \times$ weight @ $d = 1690$ km ($\sim 560 R_{\text{sch}}$)

- **torn apart**
  - before getting anywhere near event horizon
  - before getting anywhere near NS surface

- **NOTE**: $s/w \propto 1/M$ : Bigger $M =$ smaller stretch
  - $S/W = 10 \times R_{\text{Sch}}$ for $M = 13,600 \ M_{\odot}$
  - $S/W = 1 \times R_{\text{Sch}}$ for $M = 43,000 \ M_{\odot}$

- can “easily” visit monster black holes
**A visit to a BH**  
approach EH, send a regular beacon

- **You see**  
  - normal time passage  
  - accelerating downward  
  - narrowing circle of outward visibility  
  - blue stars, distorted sky  
  - hole above closes as you cross EH  
  - all views lead into BH

- **Companion sees**  
  - beacon signals further and further apart  
  - you falling slower  
  - beacon weakens, reddens  
  - eventually frozen, reddened (no signals)

**Black Holes don’t live forever**

- **Heisenberg:**  
  \[ \Delta E \times \Delta t \geq \text{tiny #} \]

  - \( E = 0 \) in the vacuum?  
    - **no!** only sure to \( \Delta E > \frac{\hbar}{\Delta t} \)  
    - for very short time, temporary (large) \( E \) possible  
    - “vacuum energy”

- **virtual particles**  
  - can pop into existence briefly  
  - matter / antimatter pairs  
  - short life - antimatter particle soon annihilates his twin

**Virtual particles near a black hole**  
energy of a BH includes gravitational potential energy  
produce virtual particle pair near (but outside) EH  
if \( -E \) enters BH, \( +E \) escapes, BH Mass goes down

- **“Hawking Radiation”**  
  \[ L = 4\pi R^2 \sigma T^4 \]
  \[ = (\Delta mc^2)/t = 4\pi [3\text{km}(M/M_{\odot})]^2 \sigma T^4 \]

  - \( T_{BH,HR} = 2\times 10^{11}K \left( \frac{M}{4\times 10^{14} \text{grams}} \right)^{-1} \)

  - \( t_{evap} = 10^{10} \text{years} \left( \frac{M}{4\times 10^{14} \text{grams}} \right)^3 \)

  - for a solar mass black hole,  
    \( T = 4\times 10^{-8} \text{ K} \) and \( t_{evap} = 1.25 \times 10^{66} \text{ years!} \)

**Black Hole evaporation**  
(and mini-black holes)

- **mini-BH as a power source**  
  - \( 4\times 10^{14}c^2 = 3.6\times 10^{35} \text{ erg} = 1 \text{ L}_\odot \) for 2 minutes!  
  - \( L = 4.1 \times 10^{15} \text{ erg/s} \)
    \[ = 4.1 \times 10^{38} \text{ W} = 410 \text{ megaW!} \] - a modest power plant!  
  - but where do you put it?!!

- **“primordial” mini black holes**  
  - \( M < 4 \times 10^{14} \text{ g} \) → would have “popped” by now  
  - \( M < 1.9 \times 10^{14} \text{ g} \) → “” “” 10⁹ yr after BB

- **Low mass BH are HOT!**  
  - \( M \sim 6.0 \times 10^{14} \text{ g} \) → \( t_{evap} = 30 \text{ Gyr}, T_{HR} = 3 \times 10^{11} \text{K} \)  
  - if common could produce a gamma-ray background  
  - none seen, so \( M_{\text{LMBH}} < 10^{-9} M_{\text{univ}} \)

- The Krennrich search for evaporating primordial BH